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Supersymmetric multiple basin attractors

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Abstract: We explain that supersymmetric attractors in general have several critical points due to the algebraic nature of the stabilization equations. We show that the critical values of the cosmological constant of the AdS_5 vacua are given by the topological (moduli-independent) formulae analogous to the entropy of the $d = 5$ supersymmetric black holes. In one-moduli case critical points with positive definite metric and gauge couplings exist under condition that the central charge changes the sign from one critical point to the other. We have found several families of $\mathbb{Z}_2$-symmetric critical points where the central charge has equal absolute values but opposite signs in two attractor points. We present examples of interpolating solutions and discuss their generic features.

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The concept of supersymmetric attractors with respect to black hole entropy was introduced in [1]. The so-called stabilization equations [2, 3] for the behaviour of the moduli near the charged extremal black holes horizon have been studied extensively during the last few years. It has been established that the supersymmetric fixed points of the theory correspond to the minimum of the central charge[1] in the physical part of the moduli space, when the metric is positive-definite [4].

The issue of uniqueness of the basin of attraction of the supersymmetric systems related to Calabi-Yau black holes of ungauged supergravity [5] and cosmological constant in $d = 5$ gauged supergravity [5] was raised in [6]. It was also shown there that the critical points of these two theories are defined by the same equations. It was emphasized in [7] that one may look for more than one basin of attraction in the context of the black hole fixed points. The situation however was not resolved in general: on the one hand one may expect that the black hole with given electric and magnetic charges in the theory with compactification on particular CY space have a unique entropy; on the other hand the stabilization equations [3, 8, 6]

$$C_{IJK}\bar{t}^J\bar{t}^K = q_I\,, \tag{1}$$

being algebraic equations, may lead to multiple solutions.[2] The reason is that one starts with the system of n quadratic equations (1) for n variables $\bar{t}^I$. They can be reduced to some higher order algebraic equation for each of the fixed moduli. For example, in case $I = 1, 2$ one has to solve quadratic equations that have 2 solutions, etc.

Indeed, solutions describing black holes with multiple basins of attraction have been studied in [9]. It was found for the 2 moduli black holes in $d = 4$ that only in one basin of attraction the scalar and vector fields have positive metric. This suggested that the argument based on the requirement of uniqueness of entropy may be correct, and only one basin of attraction is physically acceptable. Recently a solution with two basins of attraction of analogous stabilization equations was found in [10] in application to anti-de Sitter space AdS_5, but the issue of positivity of vector field metric was not analyzed there. As a result, the possibility to have physically acceptable black hole or AdS_5 configurations with multiple basins of attraction remained open.

Recent developments with AdS/CFT correspondence [11] and BPS domain walls [12]–[18] stimulated us to clarify the situation with the multiple basins of attraction for supersymmetric attractors. The purpose of this paper is to establish the conditions under which supersymmetric attractors may have more than one physically acceptable basin of attraction. We will also study the configurations interpolating between two basins.

[1] The world central charge here is used for the central charge of the supersymmetry algebra as explained in [3].

[2] We use the notations of [6], where $\mathcal{V} \equiv C_{IJK}t^it^Jt^K = 1$ and n fields $t^I, I = 1, \ldots, n$ are functions of $n-1$ independent moduli $\phi^i, i = 1, \ldots, n-1$.

The reason why these two completely different phenomena, $d = 5$ CY black holes and the cosmological constant of AdS_5 space, may be treated simultaneously *at the supersymmetric critical points* was explained in [6]. It is important to stress here however that out of critical points the systems are different: the main difference is that the term with $(\partial_i Z)^2$ enters with the opposite sign in the black hole potential and in the gauge theory potential, as shown below.

To find the black hole entropy we are looking for the supersymmetric critical points of the black hole potential [4], which for $d = 5$ ungauged $N = 2$ supergravity is given by $V = Z^2 + \frac{3}{2} g^{ij} \partial_i Z \partial_j Z$. Here the central charge $Z = t^I q_I$ depends on *real* moduli t^I and on electric charges of the black hole q_I. The supersymmetric critical points are at $Z_{,i} = 0$, where $\partial_i V = 4Z\, \partial_i Z - \sqrt{6}\; T_{ijk} \partial^j Z \partial^k Z = 0$, where T_{ijk} is a function of moduli and C_{IJK}. At the supersymmetric critical point of the central charge, which is also a supersymmetric critical point of the black hole potential, the value of the potential defines the BPS mass and the black hole entropy:

$$M_{BPS}^2 = |Z|^2, \qquad (M_{BPS}^2)_{cr} = |Z|_{cr}^2 (C_{IJK}, q_I) = |V_{cr}| \quad \text{at } Z_{,i} = 0\,. \tag{2}$$

In the supersymmetric critical points the second derivative of the black hole potential is proportional to the metric on the moduli space: $V_{,i;j} = \frac{8}{3} g_{ij} V_{\rm cr}$ at $\partial_i V = \partial_i Z = 0$. For the positive moduli space metric the potential V has a local minimum whenever the stabilization equation has a solution with non-vanishing central charge. When the potential is non-zero at the critical point, it defines the black hole entropy. The entropy $S = \frac{\pi^2}{12} \tilde{S}$ is a function of the critical value of the BPS mass

$$\tilde{S} = (M_{BPS})_{cr}^{3/2} = |Z|_{cr}^{3/2} (C_{IJK}, q_I) = |V_{cr}|^{3/4}\,. \tag{3}$$

Now consider the supersymmetric critical points of the potential of the $U(1)$ gauged $N = 2$ $d = 5$ supergravity. The size of the AdS_5 throat is defined by the extrema of the gauged supergravity potential [5, 6]. The relevant potential is equal to $-6P$, where $P = Z^2 - \frac{3}{4} g^{ij} \partial_i Z \partial_j Z$. In the context of gauged supergravity the central charge Z is a moduli-dependent combination of gravitino and gaugino charges V^I, which is defined by $Z = t^I V_I$, where V_I is the charge defining the gravitino–gravitino–vector and gaugino–gaugino–vector interactions. The critical points of the P are given by $\partial_i P = Z\, \partial_i Z + \sqrt{3/2}\; T_{ijk} \partial^j Z \partial^k Z = 0$, and as before the supersymmetric critical points of the central charge are also the supersymmetric critical points of the potential. The value of the potential P at the critical point is given by the square of the BPS mass as a function of C_{IJK} and V_I:

$$M_{BPS}^2 = |Z|^2, \qquad (M_{BPS}^2)_{cr} = |Z|_{cr}^2 (C_{IJK}, V_I) = |P_{cr}| \quad \text{at } Z_{,i} = 0\,. \tag{4}$$

The AdS_5 vacua are solutions of this theory with unbroken supersymmetry. The cosmological constant of the relevant AdS_5 space coincides with the critical value of the BPS mass extremized in the moduli space. At the supersymmetric critical points, where $Z_{,i}=0$, one finds

$$\Lambda_{AdS_5} = -6|t^I V_I|^2 = -6|Z_{\rm cr}(V_I, C_{IJK})|^2 = -6M^2_{BPS} \qquad \text{at } Z_{,i}=0\,. \tag{5}$$

In the supersymmetric critical points the second derivative of the potential is proportional to the metric on the moduli space: $-(P)_{,i;j} = -\partial_i\partial_i P = -\frac{2}{3}g_{ij}(Z^2)_{\rm cr}$ at $\partial_i P = \partial_i Z = 0$. For the positive moduli space metric the potential $-P$ has a maximum whenever the stabilization equation has a solution. When the potential is non-zero at the critical point, it defines the cosmological constant. Thus, as explained in [6, section 4.2] the critical points of the BPS mass of ungauged supergravity depending on the electric charges of the black hole solutions q_I have to be replaced by the gravitino–gaugino charges (FI terms) V_I to find the critical points of the BPS mass of the gauged theory. In ungauged theory one finds the black hole entropy from the BPS mass, in the gauged theory one finds the cosmological constant. Thus as a function of gravitino charges V_I and CY intersection numbers C_{IJK}, the cosmological constant at the supersymmetric critical point is given by the same topological formula, which defines the entropy of the extreme supersymmetric black holes: the value of the cosmological constant is moduli-independent and depends only on V_I and C_{IJK}.

Thus all previous studies of CY black hole entropy may be used for understanding the AdS_5 vacua. In what follows we will focus our attention on the issue of non-uniqueness of supersymmetric critical points of this theory.

Simultaneously with the study of the critical points of the central charge, one has to verify that at the given critical point some natural physical conditions are satisfied. We will try to find fixed points where both the metric of the moduli space $g_{ij} = -3C_{IJK}t^I t^J_{,i} t^K_{,k}$ and the metric of the vector space (the gauge coupling matrix) $\tilde G_{IJ} = -\partial_I\partial_J(\ln\mathcal{V})|_{\mathcal{V}=1}$ are positive.[3]

In some cases these conditions are sufficient to guarantee the uniqueness of the critical point for the black hole entropy. Such cases give examples of supersymmetric attractors with one basin of attraction. In some other cases specified by different values of the intersection numbers C_{IJK} and charges q_I or V_I more than one critical point satisfying physical conditions may be available. Particularly in the case of many moduli when the stabilization equations are higher order algebraic equations, one may expect to find several critical points consistent with physical requirements. In what follows we will present conditions for multi basin attractors, give examples and discuss the conceptual issues associated with such systems.

[3]Negative sign of kinetic terms in the lagrangean usually leads to vacuum instability. However, if one simultaneously changes the sign of kinetic and potential energy of some fields and if these fields live only in a part of the universe different from ours (or on a different brane) then one can avoid instabilities. This possibility deserves investigation because it may provide an explanation of the vanishing of the cosmological constant [19].

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We start with an example for $I = 1, 2$ when the algebraic system common for $d = 4$ and $d = 5$ theories has a general solution describing $d = 4$ and $d = 5$ black holes and AdS_5 vacua. In this example the issue of the uniqueness of the $d = 4$ black hole entropy and the possibility of the non-uniqueness of the entropy of $d = 5$ black holes and the critical points of the AdS_5 vacua will be clarified. One may hope to learn some lessons from this simple system which may help to understand the theories with many moduli.

Consider the simple case of $I = 1, 2$ and generic C_{IJK} and V_I. We choose

$$C_{111} = a\,, \qquad C_{112} = b\,, \qquad C_{122} = c\,, \qquad C_{222} = d\,, \tag{6}$$

and define $\bar{t}^1 \equiv x$ and $\bar{t}^2 \equiv y$. The stabilization equations consist of a system of two quadratic equations for two variables:

$$\begin{aligned} ax^2 + 2bxy + cy^2 &= V_1\,, \\ bx^2 + 2cxy + dy^2 &= V_2\,. \end{aligned} \tag{7}$$

We introduce the following notations[4]

$$\begin{aligned} M \equiv c^2 - bd\,, \qquad N \equiv b^2 - ac\,, \qquad L \equiv ad - bc\,, \\ \mathcal{D} \equiv (MV_1^2 + NV_2^2 + LV_1V_2)\,. \end{aligned} \tag{8}$$

In the context of black holes and cosmological constant we also introduce

$$E \equiv cq_1 - bq_2\,, \qquad F \equiv dq_1 - cq_2\,, \tag{9}$$

and

$$E \equiv cV_1 - bV_2\,, \qquad F \equiv dV_1 - cV_2\,, \tag{10}$$

respectively.

The metric of the moduli space is required to be positive everywhere in the moduli space. This leads to a requirement that

$$L^2 - 4MN < 0\,, \qquad M > 0\,, \qquad N > 0\,. \tag{11}$$

It follows from the fact that the expression

$$M\phi^2 - L\phi + N > 0 \tag{12}$$

has to be positive for all values of real $\phi = y/x$. This is a natural condition for the physical theory. In addition it provides the condition that the critical points

[4]We assume that $M \neq 0$ and $L^2 \neq 4MN$.

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are local minima of the black hole potential and local maxima of the gauge theory potential [4]. We exclude xy and y^2 in favour of x^2:

$$2xy = -\frac{F}{M} + \frac{Lx^2}{M}\,, \tag{13}$$

$$y^2 = \frac{E}{M} + \frac{Nx^2}{M}\,. \tag{14}$$

The system of equations can now be reduced to the quadratic equation for the variable x^2. The solution of this quadratic equation is:

$$x^2_\pm = \frac{FL + 2EM}{L^2 - 4MN} \pm \frac{\sqrt{4M^2\mathcal{D}}}{L^2 - 4MN}\,, \tag{15}$$

and $(xy)_\pm$ and $y^2_\pm$ are given by (13) and (14) in terms of $x^2_\pm$, respectively. Thus we have two sets of solutions of stabilization equations. Let us call them a + critical point and a − critical point for the x^2_+, y^2_+, xy_+ solutions and x^2_-, y^2_-, xy_- solutions, respectively. The condition for the existence of these solutions is that

$$\mathcal{D} \equiv MV_1^2 + NV_2^2 + LV_1V_2 > 0\,. \tag{16}$$

One can verify that this condition is satisfied if the moduli space metric is positive, i.e. eq. (11) is satisfied. Notice that $L^2 - 4MN < 0$ and $M > 0$, and therefore the second term in $x^2_\pm$ is always negative:

$$x^2_+ - x^2_- = 2\frac{\sqrt{4M^2\mathcal{D}}}{L^2 - 4MN} < 0\,. \tag{17}$$

The same situation takes place for the critical values of $y^2_\pm$:

$$y^2_\pm = \frac{E}{M} + \frac{Nx^2_\pm}{M} = -\frac{HL + 2EN}{L^2 - 4MN} \pm \frac{\sqrt{4N^2\mathcal{D}}}{L^2 - 4MN}\,, \tag{18}$$

where $H = bq_1 - aq_2$. Here we also find that

$$y^2_+ - y^2_- = 2\frac{\sqrt{4N^2\mathcal{D}}}{L^2 - 4MN} < 0\,. \tag{19}$$

This means that we may look for the situation when x^2_+ and y^2_+ are negative and x^2_- and y^2_- are positive. This would mean that x_+ and y_+ are imaginary and x_- and y_- are real. This is consistent with the fact that our original variables $t^I = \bar{t}^I/\sqrt{Z}$ are real if

$$Z_+ < 0\,, \qquad Z_- > 0\,. \tag{20}$$

The critical values of the cube of the central charge are given by the following expression (in the black hole case)

$$\begin{aligned} Z_{cr}(q)^3_\pm &= (t^I_\pm q_I)^3 = (\bar{t}^I_\pm q_I)^2 = (xq_1 + yq_2)^2_\pm \\ &= x^2_\pm q_1^2 + y^2_\pm q_2^2 + 2xy_\pm q_1 q_2\,. \end{aligned} \tag{21}$$

Here again we see that it is consistent to have $Z_+ < 0$ and $Z_- > 0$ for the two critical points. For the cosmological constant case, there is an analogous expression for the central charge in terms of V_I instead of q_I.

The critical values of the black hole entropy and the critical values of the cosmological constant are given by analogous formulae. By substitution of the critical values of $x^2_\pm$, $y^2_\pm$, $xy_\pm$ we get two critical values of the central charge, entropy and cosmological constant:

$$\begin{aligned} Z_{cr}(q)^3_\pm &= -\frac{q_2(dq_1^2 + bq_2^2 - 2cq_1q_2)}{M} + \mathcal{D}\left[\frac{FL + 2EM \pm \sqrt{4M^2\mathcal{D}}}{M(L^2 - 4MN)}\right], \\ \tilde{S}_\pm &= |Z_{cr}(q_I)_\pm|^{3/2}, \qquad (\Lambda_{adS_5})_\pm = -g^2|Z_{cr}(V_I)_\pm|^2 . \end{aligned} \tag{22}$$

Now we have to study the values of vector space metric (gauge couplings) at the critical points:

$$\hat{G}_{IJ} = -\frac{Z^2}{9}\partial_I\partial_J(\ln\mathcal{V})|_{\mathcal{V}=1} = V_IV_J - \frac{2}{3}C_{IJK}\bar{t}^K\bar{t}^LV_L . \tag{23}$$

A tedious calculation, analogous to that performed in [9] with respect to $d = 4$ black holes and moduli metric, allows us to find the generic expression for the determinant of the gauge coupling matrix at the critical point:

$$(\det\hat{G}_{IJ})_\pm = \mp\frac{2}{9}Z^3_\pm\sqrt{\mathcal{D}} > 0 \qquad \text{for M} > 0 . \tag{24}$$

For black holes $Z = q_1x + q_2y$ and for the cosmological constant it equals $Z = V_1x + V_2y$.

Now we see that one may be able to justify the existence of both critical points as possibly acceptable under the following conditions: We have to require that *at the $+$ critical point the critical value of the central charge Z_+ is negative and at the $-$ critical point the critical value of the central charge Z_- is positive.* In such case the determinant of gauge couplings is positive at both critical points. But this is precisely the condition obtained from the consistent solution of the attractor equations as shown in (20)! In such cases the determinant of the gauge coupling matrix is positive at both critical points.

Note that we still have to find out whether the eigenvalues of $\hat{G}_{IJ}$ are actually positive. It may happen that the determinant is positive but the eigenvalues are both negative. Fortunately a completely general treatment is possible in the simple 1-moduli case. First we note that $\det\hat{G}_{IJ} = \hat{G}_{11}\hat{G}_{22} - \hat{G}^2_{12}$ and therefore for all cases where we have established that $\det\hat{G}_{IJ} > 0$ we find that

$$\hat{G}_{11}\hat{G}_{22} > \hat{G}^2_{12} > 0 . \tag{25}$$

Thus for all cases at hand if we can verify that e.g. $\hat{G}_{11}$ is positive, this will guarantee that also $\hat{G}_{22}$ is positive and that the total gauge coupling matrix is positive-definite.

To derive the critical values of G_{11} we start with (23) and get

$$\hat{G}_{11} = \frac{1}{3}\left(q_1^2 \mp 2y_{\pm}^2\sqrt{\mathcal{D}}\right). \tag{26}$$

Now we can provide a generic answer to the problem of the positivity of the vector metric. Both critical points are locally stable iff $Z_+ < 0$ and $Z_- > 0$, as was already required by the positivity of the det of the vector metric. The same condition provides the positivity of $\hat{G}_{11}$ since the first term in (26) is always positive and the second one is also positive, as follows from eqs. (19) and (20). Thus *the moduli space metric and the gauge coupling matrix are both positive definite iff*

$$L^2 - 4MN < 0\,, \qquad M > 0\,, \qquad N > 0\,, \qquad Z_+ < 0\,, \qquad Z_- > 0\,, \tag{27}$$

and one can claim that there are two *critical points that form the stable local minima of the entropy of $d = 5$ black holes and of the absolute value of the cosmological constant of the AdS_5 vacua.*

These are generic conditions in the one-moduli problem under which two physically acceptable critical points are possible. One can give many numerical examples of cases where these conditions are satisfied. Consider for example the case suggested in [10] which in our notation is $a = 0$, $b = 2$, $c = 2.8\sqrt{3}$, $d = 6$, $V_1 = 1$, $V_2 = -0.6\sqrt{3}$. It has been verified in [10] that the moduli space metric is positive. However the vector space metric was not studied. As it is now clear from our studies, the vector space metric for this example is indeed positive since the central charge has an opposite sign in two critical points.

An interesting comment can be made about the properties of the excitations around the two gauged theory vacua. The gravitino mass near one critical point is positive $M_{-grav} = Z_- > 0$ and the one near the second critical point is negative $M_{+grav} = Z_+ < 0$ since its value at each critical point is the value of the central charge. The issue of the positive versus negative fermion mass in $d = 5$ was discussed in [20], where it was noticed that both the positive mass theory and the negative mass theory may exist. The relations between these theories include a change of γ^μ matrices into $-\gamma^\mu$, as well as of the representation of the little part SO(4) of the Lorentz group. Interestingly, we need both versions of the theory to make acceptable not only the vacuum state but also the excitations around each vacuum.

The conditions for more than one basin of attraction can be easily violated, in which case the system has only one attractor point. For example we may have some C_{IJK} and q_I (or V_I) for which either the solution is unique or the central charge has the same sign at both critical points. This would mean that one of the critical points is unstable with respect to the generation of vector fields and has to be excluded.

Typically, the values of Z^2 at two critical points is different. An interesting question therefore here is: Is the solution with $Z_+ < 0$ and $Z_- > 0$ and $|Z_+| = |Z_-|$

possible? The equation for our parameters specifying this case is:

$$\mathcal{A} \equiv -V_2(dV_1^2 + bV_2^2 - 2cV_1V_2) + \mathcal{D}\left[\frac{FL + 2EM}{(L^2 - 4MN)}\right] = 0\,. \tag{28}$$

We have found 3 families of solutions for such configurations. We take the following set of parameters $a = 0$, $b = 1/3$, $d = 1$, $V_1 = 1$ and we do not specify c and V_2, except that $c^2 > 4/9$. Equation $\mathcal{A} = 0$ has three types of solutions: $V_2 = \frac{3}{2}c$ or $V_2 = \frac{3}{2}[c \pm \sqrt{9c^2 - 4}]$. The physical conditions of the positivity of the moduli space metric and of the gauge coupling matrix are satisfied for any of these solutions, for all $c^2 > 4/9$. For example, $a = 0$, $b = 1/3$, $c = 4/3$, $d = 1$, $V_1 = 1$, $V_2 = 2$ gives a consistent set.

Having established the existence of two supersymmetric critical points describing the AdS_5 vacua some of which may have equal cosmological constant we would like to find also the interpolating domain wall solution. Here we follow [15, 16] and take an ansatz

$$ds^2 = e^{2A(r)}dx^\mu dx^\nu \eta_{\mu\nu} + dr^2 = U^2 dx^\mu dx^\nu \eta_{\mu\nu} + \frac{1}{(\partial_r A)^2}\frac{dU^2}{U^2}\,, \tag{29}$$

where $U = e^A$. At the critical points where $(\partial_r A)^2_{cr} = Z^2_{cr}$ the geometry is an AdS_5 space with a cosmological constant $\Lambda = -6Z^2_{cr}$. The equations of motion of the gauged supergravity describing a domain wall can be derived from the energy functional:

$$E = \frac{1}{2}\int_{\infty}^{+\infty} dr e^{4A}\left\{[g^{1/2}\phi' \mp 3Z_{,\phi}g^{-1/2}]^2 - 12[A' \pm Z]^2 \pm 3[e^4 AZ]^{+\infty}_{-\infty}\right\}. \tag{30}$$

Here $\phi' \equiv \partial_r\phi, A' \equiv \partial_r A$. An analogous expression was also presented in [15, 17] where it was also noticed that as different from the standard BPS situation one of the squares enters with the negative sign. Our energy functional has a non-trivial moduli space metric $g = g_{\phi\phi}$ which is absent in [15, 16, 17]. This term is important because it provides a possibility to obtain a smooth supersymmetric solution for $\phi(r)$ interpolating between the two different vacua. The first order equations of motion of the gauged supergravity which admit Killing spinors are [10, 15, 16, 17]

$$\phi'(r) = \pm 3g^{-1}Z_{,\phi}\,, \qquad A' = \mp Z\,. \tag{31}$$

We solved these equations for a wide variety of parameters which allow existence of two attractors with equal values of the cosmological constant but opposite values of Z, in a hope to find a domain wall solution of Randall-Sundrum type, with $A \propto -|r|$ at large $|r|$. However, instead of that we always found solutions[5] with $A \propto |r|$, see one of these solutions presented in figure 1.

[5] According to eq. (29), the solutions with $A \sim |r|$ tend to the large U, i.e. to UV, whereas those with $A \sim -|r|$ tend to small U, i.e. to IR.

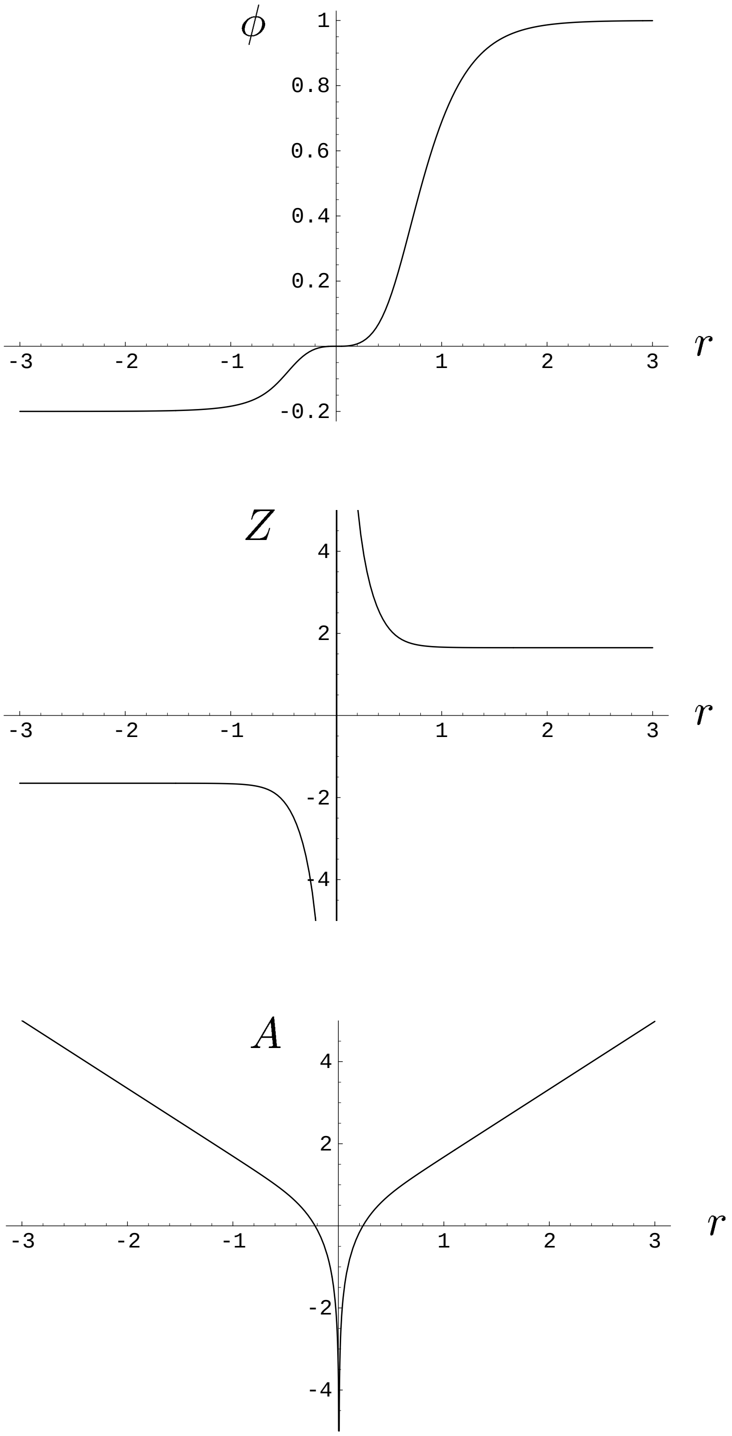

Figure 1: A solution for the scalar field ϕ interpolating between two different vacua with equal values of $|Z|$. Note that $\phi(r)$ is nonsingular because of the vanishing of g^{-1} at $\phi = 0$, whereas Z and A are singular: $Z \sim r^{-1}$ and $A(r) \sim \log |r|$ at $|r| \to 0$. At large r the function $A(r)$ grows as $|r|$ rather than decreases as $-|r|$. This is a general property of interpolating solutions in our class of models.

One can show that this is a general result, which follows from the fact that, according to [6], one has $\partial_i \partial_j Z = \frac{2}{3} g_{ij} Z$ at the supersymmetric critical point. Consider, for example, the case $\phi'(r) = -3g^{-1} Z_{,\phi}$ in eq. (31). The scalar field trajectory can approach the critical value at large r either from below or from above. The one from above with the negative derivative of ϕ requires that $Z_{,\phi} > 0$ above the attractor. The one from below requires that $Z_{,\phi} < 0$ below the attractor, whereas the attractor point corresponds to $Z_{,\phi} = 0$, therefore the second derivative of the central charge is positive, $Z_{,\phi\phi} > 0$. Since we consider the situations where $g_{ij} >$ in the attractor, equation $\partial_i \partial_j Z = \frac{2}{3} g_{ij} Z$ implies that the right critical point has a positive central charge Z. Then equation $A' = +Z$ implies that A' approaches a constant positive value at large positive r, i.e. $A \sim |Z_{cr} r| + const$, see figure 1. If we would have used the second pair of equations, staring with $\phi'(r) = 3g^{-1} Z_{,\phi}$ we would find that at the right critical point Z is negative, but in this case $A' = -Z$ and again $A' > 0$. The case when $\partial_i \partial_j Z = \frac{2}{3} g_{ij} Z = 0$ would lead to $A' = 0$ and is also ruled out. Thus it follows from supersymmetry that the interpolating solution which admits Killing spinors at large positive r behaves as $A \sim |Z_{cr} r| + const$ at large $|r|$. This is not an interpolating domain wall of [13] as was already observed in [10].

In [17] a non-supersymmetric choice of Z was suggested in the framework of supergravity equations. In this way a smooth solution modeling branes with the desired asymptotic of the interpolating solution was obtained. One can verify that the choice of the function Z in [17] is such that $Z_{,\phi\phi} \sim -Z$ at the critical points $Z_{,\phi} = 0$. This condition cannot be valid in a supersymmetric theory where $\partial_i \partial_j Z = \frac{2}{3} g_{ij} Z$ at the critical points and the moduli space metric is positive. This example confirms that solutions $A \propto -|r|$ require violation of supersymmetry in this class of theories.

Until now we were looking only for supersymmetric interpolating solutions, and found that they do not behave as $A \propto -|r|$. One may wonder whether one can find more general, non-supersymmetric interpolating solutions with the desirable asymptotic $A \propto -|r|$. The answer to this question is also negative. Indeed, the relevant equation of motion for the interpolating scalars in the background metric is

$$\phi'' + \left(4A' + \frac{g_{,\phi}}{g} \phi'\right) \phi' + 6g^{-1} P_{,\phi} = 0\,, \tag{32}$$

where at the critical points $P_{,\phi\phi}$ is positive. Let us assume that the solution of this equation asymptotically approaches an attractor point ϕ_{cr} at large $r > 0$, so that g and $g_{,\phi}$ become constant, A' becomes negative constant, and ϕ' gradually vanishes at large r. Then the deviation $\delta\phi$ of the field ϕ from its asymptotic value ϕ_{cr} at large $|r|$ satisfies the following equation:

$$\delta\phi'' - 4|A'|\delta\phi' = -6|g^{-1} P_{,\phi\phi}|\delta\phi\,. \tag{33}$$

This is equation for a harmonic oscillator with a negative friction term $-|A'|\delta\phi'$. Solutions of this equation describe oscillations of $\delta\phi$ with amplitude blowing up at large $|r|$, which contradicts our assumptions. This argument shows that there are no interpolating solutions in our theory with $A \propto -|r|$ at large $|r|$. This conclusion remains valid even if one relaxes our assumption that the scalar field metric is positive and considers domains with $g < 0$.

Finally, we would like to note that even though we called the field configuration shown in figure 1 "interpolating solution", its physical interpretation requires further investigation. Indeed, even though the solution for the scalar field ϕ smoothly interpolates between the two attractor solutions, the function $A(r)$ is singular. It behaves as $\log|r|$ at $|r| \to 0$. Metric near the domain wall is given by

$$ds^2 = r^2 dx^\mu dx^\nu \eta_{\mu\nu} + dr^2\,. \tag{34}$$

This implies the existence of the curvature singularity at $r = 0$, which separates the universe into two parts corresponding to the two different attractors.

We would like to point out that the existence of several basins of attraction in dynamical systems in general is quite common. Typically the system is attracted to the nearest attractor point after it reaches a given basin of attraction. The new result established in this paper is that there are conditions when more than one critical points in supersymmetric attractors are physically acceptable, i.e. the moduli space metric and the gauge couplings are positive. The system may be at some initial value of a moduli either from one side of the discontinuity of the moduli space metric or on the other side. This gives a precise definition of the basin of attraction. Note that the value of the entropy (or the value of the cosmological constant) in general is different for two critical points under discussion:

$$\delta\tilde{S} = \left|\mathcal{A} + 2\frac{\mathcal{D}^{3/2}}{(L^2 - 4MN)}\right|^{1/2} - \left|\mathcal{A} - 2\frac{\mathcal{D}^{3/2}}{(L^2 - 4MN)}\right|^{1/2}\,. \tag{35}$$

Thus, with respect to black holes, our analysis seems to develop and confirm the idea suggested by Moore [7] that the black holes (in $d = 5$, under some conditions specified in this paper) may represent a multiple attractor system. To specify a black hole one has to specify not only the charges and the prepotential but also the attractor point, defined by the values of moduli at some distance from the horizon. However, to fully understand the issue of the possible non-uniqueness of the black hole entropy one has to study the black hole solutions and not only the critical points. We hope to investigate this in the future.

With respect to AdS_5, we have shown that the critical points for the cosmological constant correspond to a multiple attractor system. One of the most interesting issues is related to the $\mathbb{Z}_2$-symmetric BPS critical points with equal values of the AdS_5 radius found in this paper when $\mathcal{A} = 0$. Our investigation of the domain wall solution in gauged supergravity [5] with one vector multiplet shows that for the non-compact 5-th dimension the asymptotic form of the interpolating solutions is always $e^{|Z_{cr} r|}$, which has an opposite sign compared to the Randall-Sundrum scenario [13, 10].

The main result of our paper is that multiple basins of attraction are possible in supersymmetric theories. We found double-attractor systems with positive scalar and vector metric in $d = 5$ one-moduli theory. We expect that in theories with many moduli in $d = 4$ as well as in $d = 5$ one may also find physically acceptable configurations with multiple basins of attraction.

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